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ENDOCHRONIC THEORY OF TRANSIENT CREEP
AND CREEP RECOVERY

by

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1. INTRODUCTION

Creep has been a topic of investigation for many decades. Meaningful results have been obtained by investigators both from the micromechanistic approach and the macroanalytical or continuum mechanics approach. A rich literature is available and references may be found in the books by Garofalo [1], Odqvist [2], and Rabotnov [3], among others.

Both the micromechanistic and continuum approaches can lead to fruitful results and each can benefit from the other. "The micromechanistic approach provides knowledge of the processes that control creep and provides guidelines for defining more clearly material properties and for designing better materials for specific applications. The macroanalytical approach can provide basic relations that are broad in scope and can lead to improved procedures for designing structures."*

The purpose of this report is to discuss further the behavior of creep in metals using the approach of continuum mechanics. Several theories have been proposed in the literature to describe creep using this approach. However, there still exists an unresolved problem related to the role played by strain-hardening during creep. This problem is important in the investigation of creep subject to variable stress.

In most investigations, the subjects of stress-strain relation and creep are treated separately, so that they do not bear direct relationship between them. In the writers' view, the aforementioned fact contributes greatly to the difficulty associated with the investigation of creep under variable stress. Fortunately, recent progress in the understanding of the strain-rate and strain-rate history effect in the theory of constitutive

* The words of Garofalo [1] are being quoted.

equation has made it possible to devise a unified approach which would bring diversified material behaviors, such as constant-strain-rate stress-strain relation, creep, and stress relaxation into a common workframe.

The present report is written with such thought in mind. This viewpoint is shared by such research workers as Valanis and Lalwani [4] and Cernocky and Krempl [5]. In this unified approach, creep is viewed as a special case of the general mechanical behavior of material. The constitutive creep equation is reduced directly from the general constitutive equation of the material under study but with the condition of constant stress imposed. Since the constant-strain-rate stress-strain relation is also reduced from the same general constitutive equation, it is evident that a correlation between the two areas of interest can be established. Thus, material constants and functions determined from the constant-strain-rate stress-strain curves will appear without alteration in the creep equation. The number of unknown parameters in the creep equation is thus greatly reduced.

In this report, Valanis' endochronic theory of viscoplasticity [6,7,8] is applied to tackle the problems associated with creep. The endochronic theory has been previously applied to investigate the creep behavior of metallic materials. Valanis and Lalwani [4] developed a nonlinear evolution equation for the thermodynamic internal state variables using the concept of absolute-reaction-rates theory. The equations are then applied to predict creep at moderately large strains under constant stress from the experimental data obtained in stress relaxation. Wei [9] investigated creep using the Gibbs free energy formulation and the stress-defined intrinsic time.

The approach taken in this investigation is different from those mentioned above, although all are within the framework of the endochronic

theory. In particular, the concept of intrinsic time introduced by Valanis [8] is employed in this investigation.

The problems of creep recovery and stress relaxation are also treated using equations derived from the same general constitutive equation by imposing appropriate constraint for each case. It is shown in this report that the theory agrees quite well with experimental results of Wang and Onat [10] for Aluminum 1100-0 at 150°C (300°F).

2. BRIEF SUMMARY OF ENDOCHRONIC THEORY OF VISCOPLASTICITY

The endochronic theory of viscoplasticity developed by Valanis [6,7] is based on the notion of intrinsic time and the thermodynamic theory of internal variables. Since most of the materials are, in general, strain history dependent, a time measure $d\zeta$ is defined such that

$$d\zeta^2 = P_{ijkl} d\epsilon_{ij} d\epsilon_{kl} + g dt^2 \quad (1)$$

where ϵ_{ij} is the strain tensor and P_{ijkl} is generally a function of ϵ_{ij} and a positive definite material tensor; g is a material function of ϵ_{ij} ; and t is the real time. In addition, a time scale $z(\zeta)$ is introduced such that $dz/d\zeta > 0$.

This concept together with the thermodynamic theory of the internal variables gives the following explicit constitutive equation for isotropic materials under small isothermal deformation:

$$\sigma_{ij} = \delta_{ij} \int_p^z \lambda(z - z') \frac{\partial \epsilon_{kk}}{\partial z'} dz' + 2 \int_0^z \mu(z - z') \frac{\partial e_{ij}}{\partial z'} dz' \quad (3)$$

where σ_{ij} is the stress tensor, e_{ij} is the deviatoric part of ϵ_{ij} ; δ_{ij} is the Kronecker's delta; and $\lambda(z)$ and $\mu(z)$ are heredity functions. But the definition of intrinsic time in equation (1) has led to difficulties in cases where the history of deformation involves unloading. Valanis [8] has since introduced a new concept of intrinsic time to overcome these difficulties. In the one-dimensional case the new intrinsic time ζ is defined as

$$d\zeta = \left| d\epsilon - k_1 \frac{d\sigma}{E_0} \right| \quad (4)$$

where k_1 is a positive scalar such that $0 \leq k_1 \leq 1$ and E_0 is the elastic modulus. Generalizing to three dimensions and r internal variables, a strain like tensor θ_{ij} is defined as

$$\theta_{ij} = \epsilon_{ij} - \phi_{ijkl} \sigma_{kl} \quad (5)$$

and

$$Q_{ij} = e_{ij} - \frac{k_1}{2\mu_0} s_{ij} \quad (6)$$

where ϕ_{ijkl} is a positive definite symmetric fourth-order tensor; Q_{ij} is the deviatoric part of θ_{ij} ; μ_0 is the shear modulus; and s_{ij} is the deviatoric part of σ_{ij} . Based on the formulation in reference [6], the response of metals in the small deformation region with an elastic hydrostatic response can be written as

$$\sigma_{kk} = 3K \epsilon_{kk} \quad (7)$$

and

$$s_{ij} = 2 \int_0^z \mu(z - z') \frac{\partial e_{ij}}{\partial z'} dz' \quad (8)$$

where K is the bulk modulus. If the function $\mu(z)$ is defined as

$$\mu(z) = \mu_0 G(z) \quad (9)$$

where $G(0) = 1$, and using the Laplace Transformation technique, equation (8) together with equations (6) and (9) reduce to

$$\varepsilon_{ij} = 2\mu_0 \int_0^z \rho(z - z') \frac{dQ_{ij}}{dz'} dz' \quad (10)$$

where $\rho(z)$ is related to $G(z)$ by the integral equation

$$\rho(z) - k_1 \int_0^z \rho(z - z') \frac{dG}{dz'} dz' = G(z) \quad (11)$$

The solution of this integral equation in the case of two internal variables* is obtained by the Laplace transform technique under the assumption of k_1 approaching to 1. The result is

$$E_0 \rho(z) = \frac{E_0}{\alpha_0} \delta(z) + E_1 e^{-\alpha z} \quad (12)$$

where α_0 , α , and E_1 are material parameters and $\delta(z)$ is the delta function.

In the case of uniaxial stress, the constitutive equation is given by

$$\sigma = E_0 \int_0^z \rho(z - z') \frac{d\theta}{dz'} dz' \quad (13)$$

where $\theta = \varepsilon - \frac{\sigma}{E_0}$ is the plastic strain, and equation (4) can be written as

$$d\varepsilon = |d\theta| \quad (14)$$

* Because of the complexity of the constitutive equation and the fact that the number of material parameters increases with the number of internal variables, it is natural to strive for the minimum number of internal variables which will suffice in the accurate and acceptable prediction of the material response. Two internal variables are used throughout this work.

The new form of the endochronic theory has recently been extended by Wu and Yip [11] to describe the strain rate and strain rate history effects of material behavior. In this connection, it was shown that the intrinsic time is governed by the relation

$$d\zeta = k(\dot{\theta}) |d\theta| \quad (15)$$

where $\dot{\theta}$ is the plastic strain rate.

By combining equations (12), (13), and (15) it was shown that yield stress σ_y can exist and is related to the strain-rate by

$$\sigma_y = \frac{\sigma_y^0}{k(\dot{\theta})} \left(\frac{d\zeta}{dz} \right) \quad (16)$$

where σ_y^0 is the initial yield stress at reference strain-rate. Making use of the expression (Ref. [6])

$$z = \frac{1}{\beta} \ln(1 + \beta\zeta) \quad (17)$$

where β is a material constant, the constant-plastic strain-rate stress-strain relation was obtained

$$\sigma = \sigma_y(1 + \beta_1\theta) + (\sigma_0 - \sigma_y)(1 + \beta_1\theta)[1 - (1 + \beta_1\theta)^{-n}] \quad (18)$$

in which

$$\beta_1 = k\beta = \frac{E_t}{\sigma_0} \quad (19)$$

$$n = \frac{\alpha}{\beta} + 1 \quad (20)$$

and σ_0 is the intercept of the asymptotic line of equation (18) with the stress axis; E_t is the tangent modulus of the aforementioned asymptotic line.

The strain-rate sensitivity function $k(\dot{\theta})$ was determined from the experimental constant-strain-rate stress-strain curves. It was found that the following function fit the data nicely for annealed aluminum in the strain-rate range of $10^{-4} \sim 10^3 \text{ s}^{-1}$.

$$k(\dot{\theta}) = 1 - \beta_s \log \left(\frac{\dot{\theta}}{\dot{\theta}_R} \right) \quad (21)$$

In equation (21), β_s is a material constant and $\dot{\theta}_R$ is a reference strain-rate. The expression (21) was also shown in the same report to be applicable to mild steel in the strain-rate range of $10^{-4} \sim 10^1 \text{ s}^{-1}$.

In Wu and Yip [11] this form of constitutive equation was also used to investigate the strain-rate history effect by means of low-high and high-low strain-rate change loading sequences.

3. CREEP OF ALUMINUM 1100-0

The constitutive equation (13) and the new definition of intrinsic time given by Eq. (15) are now applied to describe the creep phenomenon. To achieve this objective, a careful control of the entire creep test is in order. In a conventional creep testing, a weight pan loaded with weights is gradually released to avoid impact on the specimen. The creep strain is then the additional strain developed over the initial strain as time elapses. The initial strain referred to is developed initially by the dead weights.

The initial strain history is usually not recorded in the creep test. Due to this nature in experimentation, any theoretical work aiming at the description of the creep curve is at best an approximation.*

In the present representation it is assumed that a creep test consists of two stages. The first stage is assumed to be a tensile testing at constant plastic strain rate $\dot{\theta}_0$. During this stage the intrinsic time increases from zero to z_0 . The second stage is the creep stage where the stress is kept constant at σ^* and the intrinsic time increases from z_0 to z as creep strain develops. Thus, equation (13) becomes

$$\begin{aligned} \sigma^* = E_0 \int_0^{z_0} \delta(z - z') \frac{d\theta}{dz'} dz' + \int_0^{z_0} E_1 e^{-\alpha(z-z')} \frac{d\theta}{dz'} dz' \\ + E_0 \int_{z_0}^z \delta(z - z') \frac{d\theta}{dz'} dz' + \int_{z_0}^z E_1 e^{-\alpha(z-z')} \frac{d\theta}{dz'} dz' \end{aligned} \quad (22)$$

in which the function $\rho(z)$ is given by equation (12).

* In the case of long term creep, the effect of the initial strain history is negligible and the stress-time history may be represented by the Heaviside step function. In the case of short time creep, the initial strain history is found to have significant effect on the subsequent creep behavior.

It should be noted that during the first stage of loading for aluminum, the following equation must be satisfied

$$z_0 = \frac{1}{\beta} \ln(1 + \beta k_0 \theta_0) \quad (23)$$

where β is a material constant that appeared in equation (17) and is determined from the quasi-static stress-strain curve; $k_0 = k(\dot{\theta}_0)$; and θ_0 is the plastic strain at the end of the first stage when $z = z_0$.

Figure 1 shows a typical initial strain history recorded during a conventional creep test. Approximating this initial strain history by a straight line, it is seen that at $z = z_0$, $d\theta/dt|_{z=z_0} \neq \dot{\theta}_0$. Since stress depends on the strain-rate and its history, the actual stress at z_0 is not the same as that calculated using equation (18) by assuming a constant $\dot{\theta}_0$. Therefore, the stress must be corrected at z_0 to agree with reality. This correction of stress will be accounted for in the subsequent discussion concerning the second stage of the creep test. It is remarked, however, that if the creep test is well-controlled so that the first stage of test is truly a constant $\dot{\theta}_0$ test. The correction in stress mentioned above is then not necessary.

Equation (22) is the constitutive creep equation which may be simplified by letting

$$R(z) = E_1 \int_{z_0}^z e^{-\alpha(z-z')} \frac{d\theta}{dz'} dz' \quad (24)$$

Thus, equation (22) reduces to

$$\sigma^* = I(z) + R(z) + \sigma_y^0 \frac{d\theta}{dz} \quad (25)$$

where

$$I(z) = C_1 e^{-\alpha z} \quad (26)$$

and

$$C_1 = \frac{1}{k_0} \left(\frac{E_1}{\alpha + \beta} \right) \left\{ e^{(\alpha + \beta)z_0} - 1 \right\} \quad (27)$$

It may be shown by differentiating (24) that

$$\frac{dR}{dz} = E_1 \frac{d\theta}{dz} - \alpha R \quad (28)$$

which leads to

$$R(z) = -e^{-Az} \left\{ \frac{E_1}{\sigma_y} \left(\frac{C_1}{A - \alpha} \right) \left[e^{(A - \alpha)z} - e^{(A - \alpha)z_0} \right] - \frac{C_2}{A} \left(e^{Az} - e^{Az_0} \right) \right\} \quad (29)$$

by use of Eq. (25). The constants appeared in the above equation are given by

$$A = \frac{E_1}{\sigma_y} + \alpha \quad (30)$$

$$C_2 = \frac{E_1}{\sigma_y} \sigma^* \quad (31)$$

During the creep process, the intrinsic time is defined by

$$d\tau = k_c(\dot{\epsilon}) |d\theta| \quad (32)$$

where $k_c(\dot{\epsilon})$ is the strain-rate sensitivity function under creep condition. The function $k_c(\dot{\epsilon})$ is different from $k(\dot{\epsilon})$ in general, where the latter function was defined in (15) under constant-strain-rate condition. Since the creep behavior depends on stress as a parameter, the function $k_c(\dot{\epsilon})$ may also depend on stress as a parameter.

Using (32), (26) and (17) Equation (25) reduces further to

$$k_c(\dot{\theta}) = \sigma_y^0 e^{\beta z} / (\sigma^* - R(z) - C_1 e^{-\alpha z}) \quad (33)$$

Given a function $k_c(\dot{\theta})$ at the strain rate range of creep, equation (33) can thus lead to the determination of a creep curve.

The strain-rate sensitivity function $k_c(\dot{\theta})$ will now be discussed. It is assumed that the functional form of equation (21) is still valid for creep. Thus,

$$k_c(\dot{\theta}) = 1 - \beta_c \ln \left(\frac{\dot{\theta}}{\dot{\theta}_R} \right) \quad (34)$$

in which $\dot{\theta}$ denotes the creep rate and the constant β_c depends on stress as a parameter. The dependence of β_c on stress should be further investigated experimentally.

The creep curves are determined by use of the Newton method of finite difference. The algorithm used in the calculation is presented in Appendix (A).

To compare the theory with experiment, two sets of experimental results are needed. They are the set of constant-strain-rate stress-strain curves and the creep curves. These two sets of data should be obtained for the same material with the same heat treatment and tested at the same constant temperature. The only experimental data that are available to the authors are those for Aluminum 1100-0. Creep tests were conducted at 150°C (300°F) by Wang and Onat [10] and the constant-strain-rate stress-strain curves were recorded by Senseny, Duffy and Hawley [12] at 25°C (298°K) and 250°C (523°K) for shear-strain rates of $2 \times 10^{-4} \text{ s}^{-1}$ and $3 \times 10^2 \text{ s}^{-1}$. To determine the material constants, a set of experimental constant-strain-rate stress-strain

curves at 150°C were estimated by linear interpolation from the curves of Ref. [12].

In the creep tests by Wang and Onat specimens were prepared from two different rods for the tests referred. Thus, different creep curves were obtained for the two groups of specimens even with the same heat treatment and same constant stress. The material properties for the two groups of specimens are therefore considered to be slightly different. The following two sets of material constant have been determined:*

<u>Specimens Group I</u>	<u>Specimens Group II</u>
$\dot{\theta}_R = 1.30 \times 10^{-5} \text{ s}^{-1}$	$\dot{\theta}_R = 1.40 \times 10^{-5} \text{ s}^{-1}$
$\dot{\theta}_O = 1.20 \times 10^{-4} \text{ s}^{-1}$	$\dot{\theta}_O = 1.20 \times 10^{-4} \text{ s}^{-1}$
$\alpha = 4.622 \times 10^2$	$\alpha = 5.714 \times 10^2$
$\beta = 4.444$	$\beta = 5.494$
$E_1 = 6.596 \times 10^3 \text{ MPa } (9.566 \times 10^5 \text{ psi})$	$E_1 = 8.154 \times 10^3 \text{ MPa } (1.183 \times 10^6 \text{ psi})$
$\beta_s = 3.643 \times 10^{-2}$	$\beta_s = 3.339 \times 10^{-2}$
$E_O = 6.895 \times 10^4 \text{ MPa } (1.0 \times 10^7 \text{ psi})$	$E = 6.895 \times 10^4 \text{ MPa } (1.0 \times 10^7 \text{ psi})$
$\sigma_y^O = 13.790 \text{ MPa } (2000 \text{ psi})$	$\sigma_y^O = 13.790 \text{ MPa } (2000 \text{ psi})$

The theoretical and experimental results are shown in Figures 2-4 for specimens Group I and in Figures 5-7 for Group II. The values for β_c and the stress correction mentioned earlier are listed in Table 1 for each stress level. The results show that good agreement has been obtained for Group I specimens. For Group II, only the three cases with intermediate stress

* The constants α , β , E , E_1 , σ_y^O , β_s and $\dot{\theta}_R$ are determined from the constant-strain-rate stress-strain curves. The strain rate $\dot{\theta}$ during the first stage of the creep test must be chosen so that θ_O and z_O are consistent with equation (23). Thus, $\dot{\theta}_O$ is the only parameter which needs to be assigned in the description of a conventional creep test.

levels have achieved reasonably good agreement using the constants listed. The agreement is not good for the two lower stress curves at small time and also for the curve with high creep stress. It should be noted that the present theoretical results have been obtained by assuming that only two internal state variables are significant. To predict the creep curves at widely varied stress levels, an additional internal variable may have to be used. Whether this is indeed the case awaits further investigation.

It may be concluded that for Aluminum 1100-0, creep depends on the effects of strain-hardening and strain-rate history. If β_c is dependent on stress as mentioned above, the proposed theory can reasonably describe the creep curves with moderately varied stress levels.

4. CREEP RECOVERY

In this section, creep recovery is discussed by means of the constitutive equation (13) with the Kernel function given by (12). The specimen is first subject to creep at stress σ^* for a duration of time and then the stress is reduced to σ^{**} . The integral in equation (13) is decomposed into four integrals each representing a stage of the creep-creep recovery phenomenon. Thus,

$$\sigma^{**} = \int_0^{z_0} + \int_{z_0}^{z_1} + \int_{z_1}^{z_1} + \int_{z_1}^z \quad (35)$$

The first two integrals on the right-hand side of (35) describe the creep stage which was discussed in the previous section. The stress is then reduced from σ^* to σ^{**} when the intrinsic time reaches z_1 . The third integral represents the instantaneous elastic response during unloading. Since the intrinsic time increases only with the plastic strain the value of this integral is zero. Finally, the fourth integral represents the recovery stage.

The first integral may be integrated to give $I(z)$. The second integral may be denoted by

$$R_1(z) = E_0 \int_{z_0}^{z_1} \rho(z - z') \frac{d\theta}{dz'} dz' \quad (36)$$

It can be found by differentiation that

$$\frac{dR_1(z)}{dz} = -\alpha R_1(z) \quad (37)$$

whose solution is

$$R_1(z) = C e^{-\alpha z} \quad (38)$$

where

$$C = -e^{-(A-\alpha)z_1} \left\{ \frac{E_1}{\sigma_y^0} \left(\frac{C_1}{A-\alpha} \right) \left[e^{(A-\alpha)z_1} - e^{(A-\alpha)z_0} \right] - \frac{C_2}{A} \left(e^{Az_1} - e^{Az_0} \right) \right\} \quad (39)$$

and

$$R_1(z_1) = R(z_1) \quad (40)$$

The fourth integral is expressed as

$$E_0 \int_{z_1}^z \rho(z - z') \frac{d\theta}{dz'} dz' = \sigma_y^0 \frac{d\theta}{dz} + R_2(z) \quad (41)$$

where

$$R_2(z) = E_1 \int_{z_1}^z e^{-\alpha(z - z')} \frac{d\theta}{dz'} dz' \quad (42)$$

Therefore, the constitutive equation for creep recovery may be rewritten as

$$\sigma^{**} = I(z) + R_1(z) + \sigma_y^0 \frac{d\theta}{dz} + R_2(z) \quad (43)$$

or

$$\frac{d\theta}{dz} = \frac{1}{\sigma_y^0} [\sigma^{**} - I(z) - R_1(z) - R_2(z)] \quad (44)$$

The function $R_2(z)$ may be found in closed form, which may be achieved by differentiating equation (42) to obtain

$$\frac{dR_2(z)}{dz} = E_1 \frac{d\theta}{dz} - \alpha R_2(z) \quad (45)$$

Upon substituting (44) into (45), the resulting equation can be integrated.

The result is

$$R_2(z) = -e^{-Az} \left\{ \frac{E_1}{\sigma_y^o} \left(\frac{C_1'}{A - \alpha} \right) \left[e^{(A-\alpha)z} - e^{(A-\alpha)z_1} \right] - \frac{C_2'}{A} \left(e^{Az} - e^{Az_1} \right) \right\} \quad (46)$$

where

$$C_1' = C_1 + C \quad (47)$$

$$C_2' = \frac{E_1}{\sigma_y^o} \sigma^{**} \quad (48)$$

and

$$R_2(z_1) = 0 \quad (49)$$

Therefore, it is obvious from equation (44) that $\frac{d\theta}{dz}$ is a function of z only if σ^* and σ^{**} are specified. Using now equations (32) and (17), equation (44) can be written as

$$k_c^{**}(\dot{\theta}) = \pm \frac{\sigma_y^o e^{\beta z}}{\sigma^{**} - I(z) - R_1(z) - R_2(z)} \quad (50)$$

in which k_c^{**} denotes the function $k_c(\dot{\theta})$ during the stage of recovery. The significance of sign "+" will be discussed later in this section. It is noted from equation (50) that k_c^{**} is a function of z .

A procedure similar to that used for creep stage is now followed. Assuming that the relation (32) holds also during the stage of recovery, the strain-rate at recovery may be expressed by

$$\dot{\epsilon}(z) = \dot{\epsilon}_R e^{(1-k^{**}(z))/\beta_c} \quad (51)$$

An algorithm of numerical calculation for creep recovery is presented in Appendix (B).

The experimental results of Wang and Onat for Aluminum 1100-0 are again used for comparison. Three examples were considered in this calculation. The first (Test I) involved a specimen crept at constant stress of 24.13 MPa (3500 psi) for 4 hours and then unloaded to a stress of 1.38 MPa (200 psi) which was far below the yield stress of 13.79 MPa (2000 psi). In this calculation, the equation (50) took the minus sign since the normal strain recovery effect was predominant at such a low stress of $\sigma^{**} = 1.38$ MPa. The normal strain recovery effect will be elaborated later in this section.

For this case, the material constants were determined by the method described in the previous sections as follows:

$$\begin{aligned} \dot{\epsilon}_O &= 1.20 \times 10^{-4} \text{ s}^{-1} \\ \dot{\epsilon}_R &= 1.40 \times 10^{-5} \text{ s}^{-1} \\ \beta &= 5.259 \\ \alpha &= 4.944 \times 10^2 \\ E_1 &= 7.06 \times 10^3 \text{ MPa } (1.024 \times 10^6 \text{ psi}) \\ \beta_c &= 3.858 \times 10^{-2} \end{aligned}$$

Both theory and experiment are shown in Fig. 8 for comparison.

The second and the third examples (Test II and III) involve specimens loaded at constant stress of 27.58 MPa (4000 psi) for one and four hours, respectively, and then unloaded to a stress of 25.86 MPa (3750 psi). In both cases, equation (50) took the positive sign since the forward creep effect was predominant at the stress specified (25.86 MPa). The details

concerning the forward creep and the recovery effects will be further discussed.

For these two cases, the material constant were determined as:

$$\dot{\epsilon}_0 = 1.20 \times 10^{-4} \text{ s}^{-1}$$

$$\dot{\epsilon}_R = 1.40 \times 10^{-5} \text{ s}^{-1}$$

$$\beta = 5.494$$

$$\alpha = 5.714 \times 10^2$$

$$E_1 = 8.16 \times 10^3 \text{ MPa} (1.183 \times 10^6 \text{ psi})$$

$$\beta_c = 3.339 \times 10^{-2}$$

Figure 9 shows the theoretical and the experimental results. It is seen that the agreement is quite good for a short time. The results also show that the amount of forward creep during the process of recovery is significantly affected by the length of creep time before unloading. For the same creep condition, the earlier the unloading takes place the greater the forward creep rate.

In Test II, the theoretical curve rises above the experimental curve four hours after unloading has taken place. Its implication is that the present theory using two internal state variables is not adequate in describing creep recovery at larger time. For better accuracy additional terms (corresponding to more internal state variables) may have to be introduced in equation (12).

The nature of creep recovery will now be further discussed. Experiments show that a critical stress σ_{cr} exists. If during creep recovery, $\sigma^{**} > \sigma_{cr}$ then forward creep will occur, i.e., the recovery curve has a positive slope. If $\sigma^{**} < \sigma_{cr}$, then normal strain recovery is found which corresponds to the curve with negative creep rate. This experimental finding has been reported

in the literature (see, for instance, Ref. [1], p. 24). In particular the experiments of Wang and Onat [10] also confirmed this observation. In the three examples mentioned earlier in this section, Test I is the case of normal strain recovery with $\sigma^{**} < \sigma_{cr}$ and Tests II and III are the cases of forward creep with $\sigma^{**} > \sigma_{cr}$.

The observed phenomena of forward creep and normal strain recovery will now be explained using the present theory. Just before unloading, the stress is σ^* and the intrinsic time is denoted by z_1^- . From equation (25), the following equation is obtained

$$\sigma^* = I(z_1^-) + R(z_1^-) + \sigma_y^0 \left. \frac{d\theta}{dz} \right|_{z_1^-} \quad (52)$$

Immediately after unloading, the stress is σ^{**} , the intrinsic time is z_1^+ , and from (43) the equation below is obtained

$$\sigma^{**} = I(z_1^+) + R_1(z_1^+) + \sigma_y^0 \left. \frac{d\theta}{dz} \right|_{z_1^+} \quad (53)$$

But $I(z)$, $R(z)$, $R_1(z)$, and $R_2(z)$ are all continuous so that at z_1 , $R_1(z_1^+) = R(z_1^-)$ and $I(z_1^+) = I(z_1^-)$. Therefore, equations (52) and (53) can be combined to yield

$$\sigma^* - \sigma^{**} = \sigma_y^0 \left. \frac{d\theta}{dz} \right|_{z_1^-} - \sigma_y^0 \left. \frac{d\theta}{dz} \right|_{z_1^+} \quad (54)$$

Furthermore, by using equations (32) and (17), it may be found that

$$\left. \frac{d\theta}{dz} \right|_{z_1^-} = \frac{\sigma_y^0}{k_c} e^{\beta z_1^-} \quad (55)$$

and

$$\left. \frac{d\theta}{dz} \right|_{z_1^+} = \pm \frac{\sigma_y^0}{k_c^{**}} e^{\beta z_1^+} \quad (56)$$

The "+" sign in (56) corresponds to the absolute value sign in equation (32). In the case of forward creep, where $\dot{\theta} > 0$, the positive sign should be used; whereas the negative sign is used when the case of normal strain recovery is being considered, i.e., when $\dot{\theta} < 0$.

Substituting equations (55) and (56) into (54), the following expression for the strain-rate sensitivity function is obtained

$$k_c^{**} = \pm 1 / \left[\frac{1}{k_c} - \frac{\sigma^* - \sigma^{**}}{\sigma_y^0 e^{\beta z_1}} \right] \quad (57)$$

In the special case of no unloading, i.e., $\sigma^* = \sigma^{**}$, equation (57) leads to $k_c^{**} = k_c$ (only the positive sign is meaningful due to positive creep rate), which of course represents the case of creep.

When at z_1^+ , equation (50) is reduced to

$$k_c^{**}(z_1^+) = \pm \frac{\sigma_y^0 e^{\beta z_1}}{\sigma^{**} - I(z_1) - R_1(z_1)} \quad (58)$$

Utilizing this equation, the following observations may be made. On the right-hand side of equation (58), the denominator is zero when $\sigma^{**} = I(z_1) + R_1(z_1)$. Therefore, a critical stress σ_{cr} may be defined so that when $\sigma^{**} = \sigma_{cr}$, $k_c^{**}(z_1^+)$ approaches to infinity, which in turn gives $\dot{\theta} = 0$ by use of equation (51). Thus, creep recovery at the critical stress σ_{cr} will result in a zero strain recovery rate at z_1 . For $z > z_1$, the rate of

recovery is governed by equation (50) in which $R_2(z) \neq 0$. For $\sigma^{**} > \sigma_{cr}$, the denominator is greater than zero, thus the plus sign must be chosen in (58) for k_c^{**} to be positive. As mentioned earlier, the plus sign corresponds to the case of forward creep. On the contrary, if $\sigma^{**} < \sigma_{cr}$, then the minus sign should be used, which of course corresponds to the case of normal strain recovery.

A further observation related to equation (58) is that in the case of forward creep, the greater the stress σ^{**} is, the smaller is k_c^{**} ; hence, the greater is the rate of forward creep. In the case of normal strain recovery, the reverse is true, i.e., the smaller the stress σ^{**} is, the smaller is k_c^{**} ; thus, the greater is the rate of recovery. The above observation agrees with Garofalo [1] and Wang and Onat [10].

5. STRESS RELAXATION

A preliminary investigation of stress relaxation is reported in this section. Due to the non-availability to the authors of the stress relaxation data for Aluminum 1100-0 at 150°C (300°F), only the theoretical results will be discussed.

To treat the subject of stress relaxation using the present theory, it is recalled that

$$d\theta = d\epsilon - \frac{d\sigma}{E_0} \quad (59)$$

During the process of stress relaxation, the total strain is kept constant. Thus,

$$\dot{\theta} = - \frac{\dot{\sigma}}{E_0} \quad (60)$$

and the plastic strain-rate is seen to be related to the rate of change of stress during stress relaxation.

The constitutive equation governing stress relaxation is again reduced from the general constitutive equation given by (13). The relaxation test is divided into two stages as in creep test. The first stage is that of the constant-strain-rate tension test and the second stage is the stage of stress relaxation.

It may be shown that the following governing equation can be reduced directly from equation (13):

$$\sigma(z) = I(z) + R(z) + \sigma_y^0 \frac{d\theta}{dz} \quad (61)$$

where $I(z)$ and $R(z)$ were previously defined in equations (26) and (24).

The symbol z_c is again used to denote the intrinsic time when the second stage of test begins.

For stress relaxation, the following equations are again obtained

$$\frac{dR(z)}{dz} = E_1 \frac{d\theta}{dz} - \alpha R \quad (62)$$

$$\frac{d\theta}{dz} = \frac{1}{\sigma_y} [\sigma(z) - I(z) - R(z)] \quad (63)$$

These equations may be combined to yield a first order differential equation in $R(z)$. The following solution for R is obtained by requiring that $R(z_0) = 0$:

$$R = -e^{-Az} \left\{ \frac{E_1}{\sigma_y} \left(\frac{C_1}{A - \alpha} \right) \left[e^{(A-\alpha)z} - e^{(A-\alpha)z_0} \right] - \int_{z_0}^z e^{Az'} g(z') dz' \right\} \quad (64)$$

where A and C_1 are defined in (30) and (27) and

$$g(z) = \frac{E_1}{\sigma_y} \sigma(z) \quad (65)$$

During the relaxation process, the intrinsic time is defined by

$$dz = k_r(\dot{\theta}) |d\theta| \quad (66)$$

where $k_r(\dot{\theta})$ is the strain-rate sensitivity function under the condition of relaxation. This function must be different from $k_c(\dot{\theta})$ which is the strain-rate sensitivity function for creep. During relaxation the total strain is kept constant and this constraint should be reflected through $k_r(\dot{\theta})$. It is anticipated that $k_c(\dot{\theta})$ and $k_r(\dot{\theta})$ are related. However, no explicit relationship between these two functions can be obtained without further experimentation.

It is assumed that the function $k_r(\dot{\theta})$ takes the following form

$$k_r(\dot{\theta}) = 1 - \beta_r \ln \left(\frac{\dot{\theta}}{\dot{\theta}_R} \right) \quad (67)$$

where β_r may depend on the magnitude of plastic strain when relaxation stage begins. The fact that the theoretical results to be presented later agree qualitatively with the usual stress relaxation curves indicates that the functional form given by (67) is reasonable.

It may be obtained from (63) and (66) that

$$k_r(z) = \frac{\sigma_y^0 e^{\beta z}}{\sigma(z) - I(z) - R(z)} \quad (68)$$

In addition, equation (60) leads to

$$\dot{\sigma}(z) = -E_0 \dot{\theta}_R e^{(1-k_r)/\beta_r} \quad (69)$$

Thus, equations (68) and (69) govern the relaxation behavior of a material.

An algorithm for the numerical calculation of the relaxation curves are given in Appendix (C). The theoretical results are shown in Fig. 10.

6. CONCLUSIONS

The endochronic theory of viscoplasticity has been applied to discuss creep, creep recovery, and stress relaxation at the small strain and short time range. The following conclusions may be drawn from this investigation:

(1) The governing constitutive equations for constant-strain-rate stress-strain behavior, creep, creep recovery, and stress relaxation have been derived by imposing appropriate constraints on the general constitutive equation (13).

(2) A set of material constants has been found which correlate strain-hardening, creep, creep recovery, and stress relaxation.

(3) The theory predicts with reasonable accuracy the creep and creep recovery behaviors at short time.

(4) The strain-rate history at prestraining stage affects the subsequent creep.

(5) A critical stress σ_{cr} has been established for creep recovery. If $\sigma^{**} > \sigma_{cr}$, forward creep will occur. If $\sigma^{**} < \sigma_{cr}$, then normal strain recovery will take place.

(6) The correlation between the strain-rate sensitivity functions $k(\dot{\theta})$, $k_c(\dot{\theta})$, and $k_r(\dot{\theta})$ cannot be obtained without further experimental investigation.

7. ACKNOWLEDGMENT

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APPENDIX (A) - Algorithm for the Computation of Creep Curves

Based on equations (33) and (34), the creep curves are computed by use of the Newton method of finite difference. The following steps are suggested for computing creep curves obtained by the conventional method of creep testing:*

- Step 1: Determine the material constants α , β , E_0 , E_1 , σ_y^0 , β_s , and $\dot{\theta}_R$ from the experimental constant-strain-rate stress-strain curves.
- Step 2: Compute the plastic strain $\theta_0 = \epsilon_0 - \sigma^*/E_0$ where ϵ_0 and σ^* are known values.
- Step 3: Try and error procedure is used to determine $\dot{\theta}_0$. For each $\dot{\theta}_0$ chosen, ζ_0 and z_0 can be computed by use of equations (21), (32), and (23). The rest of the procedures are then followed to compute the creep curve. However, if the computed creep curve does not agree with the experimental curve, a new value of $\dot{\theta}_0$ is assumed.
- Step 4: For each step, compute $k_c^{(n)}(\dot{\theta}^{(n)})$ by substituting $z^{(n)}$ and σ^* into equation (33), where n starts from zero and is a positive integer.
- Step 5: Use equation (34) to compute $\dot{\theta}^{(n)}$.
- Step 6: Choose a reasonably small step size Δt and compute $\Delta\theta^{(n)} = \dot{\theta}^{(n)} \Delta t$ and $\Delta\zeta^{(n)} = k_c^{(n)} \Delta\theta^{(n)}$.
- Step 7. Set $t^{(0)} = 0$, $\zeta^{(0)} = \zeta_0$, $z^{(0)} = z_0$, then compute
- $$t^{(n+1)} = t^{(n)} + \Delta t$$

For well controlled creep test, i.e., $\dot{\theta}_0 = \text{constant}$, the quantities $\dot{\theta}_0$ and θ_0 are known. The intrinsic time z_0 and the stress σ^ can be calculated from equations (21), (23), and (18). Noting that the total strain at the beginning of creep stage is $\epsilon_0 = \theta_0 + \sigma^*/E_0$, the computation may proceed by following steps 4 through 8.

$$\theta^{(n+1)} = \theta^{(n)} + \Delta\theta^{(n)}$$

$$\varepsilon^{(n+1)} = \theta^{(n+1)} + \varepsilon^{(n)}$$

$$\zeta^{(n+1)} = \zeta^{(n)} + \Delta\zeta^{(n)}$$

$$z^{(n+1)} = z^{(n)} + \frac{1}{\beta} \ln \left[\frac{1 + \beta\zeta^{(n+1)}}{1 + \beta\zeta^{(n)}} \right]$$

The $\varepsilon^{(n)}$ vs. $t^{(n)}$ profile is the computed creep curve.

Step 8: Specify a terminal time T . If $t^{(n+1)}$ is greater than T , stop the computation. Otherwise, return to Step 4 with $t^{(n)}$, $\theta^{(n)}$, $\varepsilon^{(n)}$, $\zeta^{(n)}$, and $z^{(n)}$ replaced by $t^{(n+1)}$, $\theta^{(n+1)}$, $\varepsilon^{(n+1)}$, $\zeta^{(n+1)}$, and $z^{(n+1)}$, respectively.

APPENDIX (B) - Algorithm for the Computation of Creep Recovery Curves

Curves of creep recovery may be computed from equations (50) and (51) using the following procedures:

- Step 1: Use all previously determined material constants.
- Step 2: Calculate the critical stress $\sigma_{cr} = I(z_1) + R_1(z_1)$.
- Step 3: Compare σ^{**} with σ_{cr} . If $\sigma^{**} > \sigma_{cr}$, use positive sign in equation (50). Otherwise, the negative sign should be used.
- Step 4: For each step, compute $k_c^{**(n)}$ by substituting $z^{(n)}$ and σ^{**} into equation (50), where n starts from zero and is a positive integer.
- Step 5: Use equation (51) to compute $\dot{\theta}^{(n)}$.
- Step 6: Choose a reasonably small step size Δt and compute $\Delta\theta^{(n)} = \dot{\theta}^{(n)}\Delta t$ and $\Delta\zeta^{(n)} = k_c^{**(n)}\Delta\theta^{(n)}$.
- Step 7: Set $t^{(0)} = t_1$, $\zeta^{(0)} = \zeta_1$, $z^{(0)} = z_1$ where t_1 is the time at the onset of recovery. Compute $\epsilon^{(0)} = \epsilon_1 - (\sigma^* - \sigma^{**})/E_0$, where $\epsilon^{(0)}$ and ϵ_1 are the recovery and creep strain at t_1 , respectively.
- Step 8: Compute $t^{(n+1)}$, $\theta^{(n+1)}$, $\epsilon^{(n+1)}$, $\zeta^{(n+1)}$, and $z^{(n+1)}$ using the equations in Step 7 of Appendix (A). The $\epsilon^{(n)}$ vs. $t^{(n)}$ profile is the computed creep recovery curve.
- Step 9: Same as Step 8 of Appendix (A).

APPENDIX (C) - Algorithm for the Computation of Stress Relaxation Curves

Equations (68) and (69) are used for the computation of the stress relaxation curves. In the computation of $R(z)$ from (64), the function $g(z)$ is assumed piece-wise linear. The following steps are followed:

- Step 1: Use same material constants as those in Appendix (A).
- Step 2: Compute θ_0 , ζ_0 , and z_0 using Steps 2 and 3 of Appendix (A).
- Step 3: When $n = 0$, set $\sigma^{(n)} = \sigma_0^r$, $\zeta^{(n)} = \zeta_0$, $z^{(n)} = z_0$, $t^{(n)} = 0$, where n starts from zero and is a positive integer, σ_0^r is the initial relaxation stress.
- Step 4: For each step, except $n = 0$, compute $R(z^{(n)})$ with the assumption of piece-wise linear $\sigma(z)$:

$$\sigma(z) = \frac{\sigma^{(n)} - \sigma^{(n-1)}}{\Delta z^{n-1}} (z - z^{(n-1)}) + \sigma^{(n-1)}$$

If $n = 0$, set $R(z^{(n)}) = 0$.

- Step 5: Compute $k_r^{(n)}$ by substituting $z^{(n)}$, $\sigma^{(n)}$, and $R(z^{(n)})$ into equation (68). n starts from zero in this computation.
- Step 6: Calculate stress rate $\dot{\sigma}^{(n)}$ from equation (69).
- Step 7: Choose a reasonably small step size Δt and compute the following:

$$\Delta \sigma^{(n)} = \dot{\sigma}^{(n)} \Delta t$$

$$\Delta \zeta^{(n)} = - \frac{k_r^{(n)} \dot{\sigma}^{(n)}}{E_0} \Delta t$$

$$\Delta z^{(n)} = \frac{\Delta \zeta^{(n)}}{1 + \beta \zeta}$$

Step 8: Compute

$$t^{(n+1)} = t^{(n)} + \Delta t^{(n)}$$

$$\sigma^{(n+1)} = \sigma^{(n)} + \Delta \sigma^{(n)}$$

$$\zeta^{(n+1)} = \zeta^{(n)} + \Delta \zeta^{(n)}$$

$$z^{(n+1)} = z^{(n)} + \Delta z^{(n)}$$

The $\sigma^{(n)}$ vs. $t^{(n)}$ profile is the computed stress relaxation curve.

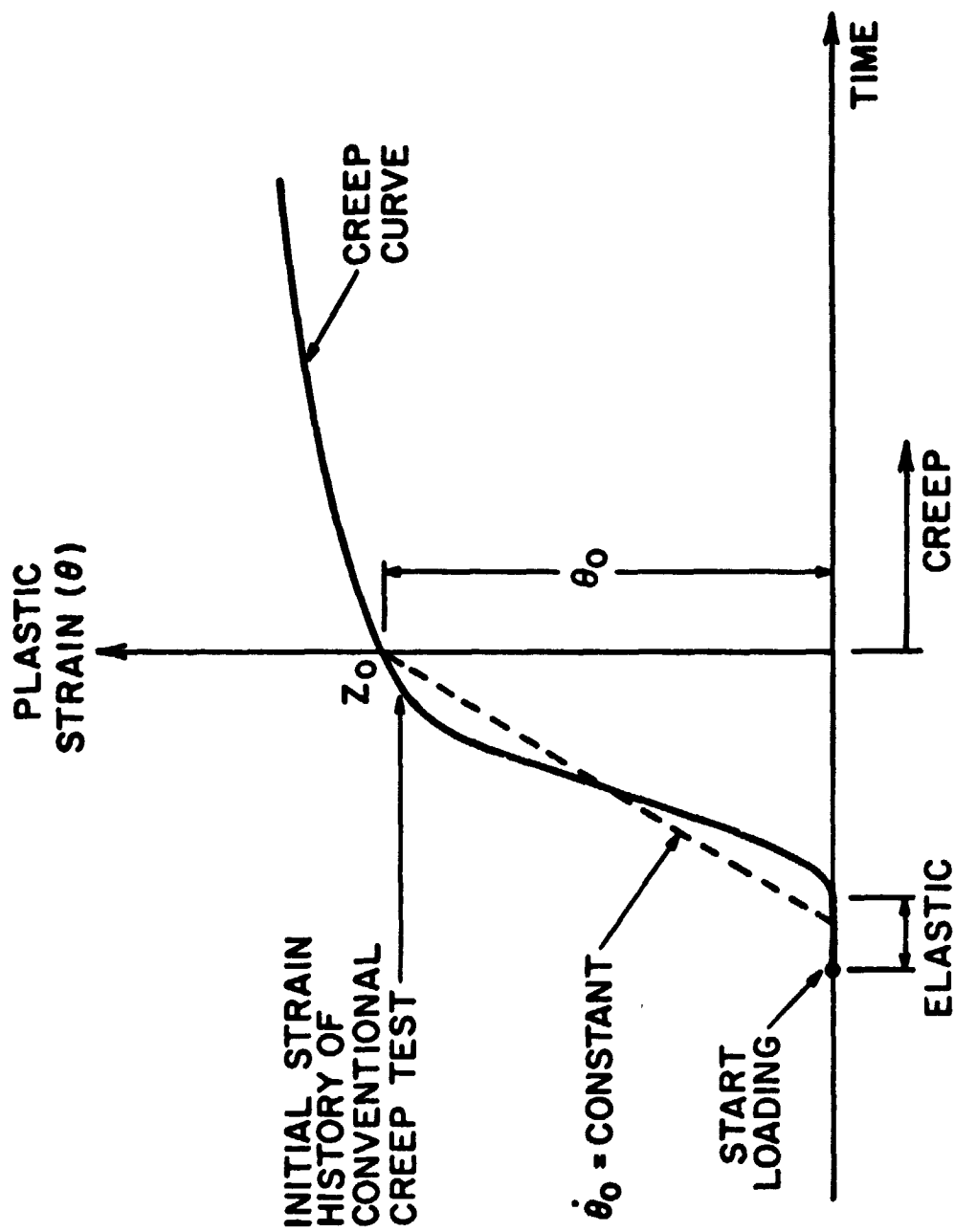
Step 9: Specify a terminal time T . If $t^{(n+1)}$ is greater than T , stop the computation. Otherwise, return to Step 4 with $t^{(n)}$, $\sigma^{(n)}$, $\zeta^{(n)}$, and $z^{(n)}$ replaced by $t^{(n+1)}$, $\sigma^{(n+1)}$, $\zeta^{(n+1)}$, $z^{(n+1)}$, respectively.

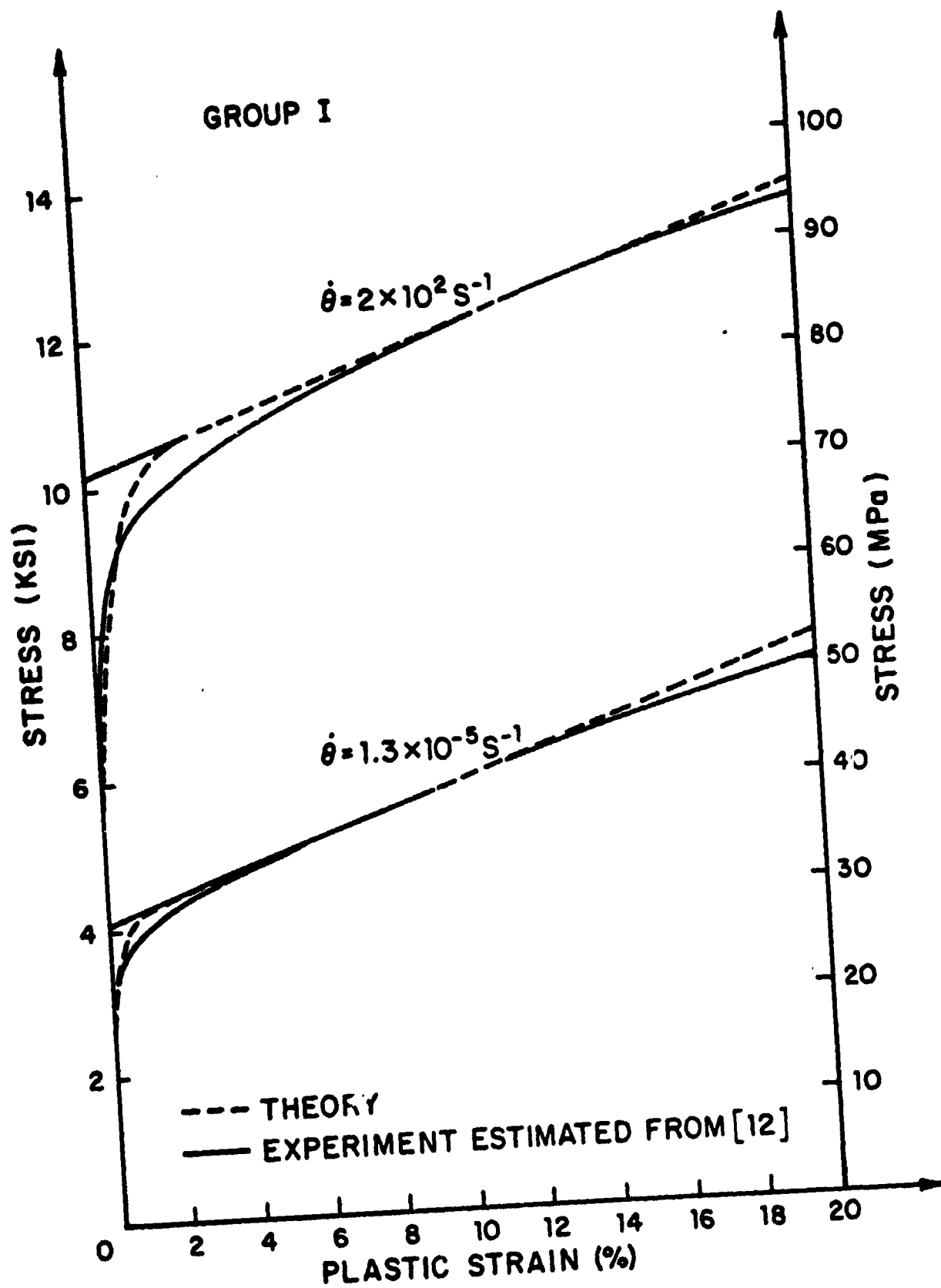
TABLE 1
Corrections in Creep for Various Creep Stresses

Specimen Group	Creep Stress MPa (psi)	β_c	Stress Correction MPa (psi)
I	24.13 (3500)	β_s	-1.990 (-288.68)
	25.86 (3750)	0.71 β_s	-1.643 (-238.34)
II	17.24 (2500)	β_s	+0.454 (+65.80)
	20.68 (3000)	β_s	-0.869 (-125.97)
	24.13 (3500)	β_s	-1.724 (-250.06)
	25.86 (3750)	0.79 β_s	-1.628 (-236.06)
	27.58 (4000)	0.43 β_s	-1.467 (-212.70)

FIGURE CAPTIONS

1. Initial Strain History in Creep Test
2. Constant-Strain-Rate Stress-Strain Curves of Aluminum 1100-0 at 150°C (300°F) - Specimens Group I
3. Strain-Rate Sensitivity Function k for Group I
4. Creep Curves for Aluminum 1100-0 at 150°C (300°F) - Specimens Group I
5. Constant-Strain-Rate Stress-Strain Curves of Aluminum 1100-0 at 150°C (300°F) - Specimens Group II
6. Strain-Rate Sensitivity Function k for Group II
7. Creep Curves for Aluminum 1100-0 at 150°C (300°F) - Specimens Group II
8. Creep Recovery Curve for Aluminum 1100-0 at 150°C (300°F) - Test I
9. Creep Recovery Curves for Aluminum 1100-0 at 150°C (300°F) - Tests II and III
10. Theoretical Stress ^{Relaxation} ~~Relations~~ Curves





GROUP I

$$k = 1 - \beta_s \ln\left(\frac{\dot{\theta}}{\dot{\theta}_R}\right)$$

$$\beta_s = 3.643 \times 10^{-2}$$

